NASA TECHNICAL TRANSLATION NASA TT F-15,188

LIMITS OF SUITABILITY OF PSEUDORANDOM SIGNALS FOR STATISTICAL TESTS

A. Gaugg, M. Seidel, H. Weinrichter

Translation of "Grenzen der Eignung von Pseudozufallssignalen für statistische Tests," Archiv für Elektronik und Übertragungstechnik, Vol. 27, January 1973, pp. 30-36

(NASA-TT-F-15188) LIMITS OF SUITABILITY OF PSEUDORANDOM SIGNALS FOR STATISTICAL TESTS (Kanner (Leo) Associates) ~~20~~ 21 p HC $3.00 CSCL 20N

N74-11968

G3/07 Unclas 22746

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 NOVEMBER 1973

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TT F-15,188		

4. Title and Subtitle	5. Report Date
LIMITS OF SUITABILITY OF PSEUDORANDOM SIGNALS FOR STATISTICAL TESTS	November 1973
	6. Performing Organization Code

7. Author(s)	8. Performing Organization Report No.
A. Gaugg, M. Seidel, H. Weinrichter Institute of Low Frequency Technology, Technische Hochschule, Vienna	10. Work Unit No.

9. Performing Organization Name and Address	11. Contract or Grant No.
Leo Kanner Associates, P.O. Box 5187, Redwood City, California 94063	NASw-2481
	13. Type of Report and Period Covered
	Translation

12. Sponsoring Agency Name and Address	14. Sponsoring Agency Code
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION, WASHINGTON, D.C. 20546	

15. Supplementary Notes

Translation of "Grenzen der Eignung von Pseudozufallssignalen für statistische Tests," Archiv für Elektronik und Übertragungstechnik, Vol. 27, January 1973, pp. 30-36.

16. Abstract The paper shows typical dependences in the pattern of m-sequences of the length 2^n-1 which take effect in particular with linear filtering. For $k<n$, filtering in the form of a summation of k consecutive bits results in no essential differences with respect to comparable genuine random signals as far as amplitude distribution and frequency of transition are concerned. For $k = n$, the frequencies of transition exhibit typical differences, depending on the principle of m-sequence generation. For $k>n$, strong asymmetries in the amplitude distribution result as well. This leads to the conclusion that, despite ideal correlation properties over the entire period duration, the individual bits of an m-sequence can only be considered as statistically independent within intervals of length n of the generating shift register.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified - Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	19	

LIMITS OF SUITABILITY OF PSEUDORANDOM SIGNALS FOR STATISTICAL TESTS

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1. Introduction

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In recent years, statistical measurement techniques have been introduced in many areas of engineering and enabled us to find an optimal solution for highly complex problems. In addition to the use of "genuine" random signals, the so-called pseudorandom signals are increasing in importance. They involve periodic signals generated according to a pre-established algorithm and which have properties similar to those of "genuine" random signals when observed over prolonged periods of time. They offer the following advantages:

1. They can be made available mostly according to a simple algorithm and used as a test signal in the solution of wide-ranging problems with minimum engineering input.

2. Their properties are clearly established by the generating principle and are exactly known.

3. They are reproducible with a high degree of accuracy even in their details, so that absolutely equal test conditions are always ensured.

The major problem in the use of pseudorandom signals lies in the question whether in each concrete case a substitute of a genuine random signal is present or whether the dependences set up by the generating principle lead to a falsification of the measurement results.

*Numbers in the margin indicate pagination in the foreign text.

In electrical engineering, binary m sequences of maximal length used in the generation of pseudorandom signals are available, since they can be easily generated by means of feedback shift registers and since they have some properties which correspond to those of a genuine random signal.

2. Known Properties of Binary Pseudorandom Sequences

Figure 1 shows the generating principle of an m sequence by means of a dual feedback shift register. For every possible register length n there are generally several possible feedback combinations which all lead to m sequences of a maximal length of $2^n - 1$ bits. More detailed discussions of the properties of m sequences can be found in the literature, e.g. [1]. At first glance, the m sequences appear to have idealized properties of a genuine random process which are being briefly summarized:

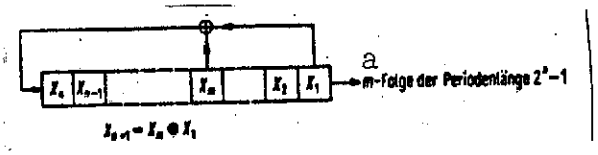

Fig. 1. Principle of generation of a binary m sequence with a dual feedback shift register.

Key: a. m sequence of period length $2^n - 1$

1. Every m sequence of period length $2^n - 1$ contains within one period almost an equal number of logical zeros and ones (exactly, one 1-bit more than 0-bits). /31

2. Of the possible 2^n combinations of zeros and ones within the shift register of length n that produces the m sequence, all combinations occur once within one period with the exception of the single combination of all zeros.

3. The number of blocks of k consecutive ones or zeros (with $k < n - 1$) is 2^{n-k-2}. In an m sequence of period $L = 2^n - 1$ there are exactly $(L + 1)/4$ blocks of zeros and ones. Of these blocks, half are of length 1, one quarter are of length 2, one

eighth of length 3, etc., and finally one block of zeros is of length n - 1 and one block of ones is of length n.

4. The autocorrelation function of the first order of the m sequence assumes only two values, namely the maximal value in the zero shift and the minimal value in all other shifts. This means that two consecutive bits are practically uncorrelated.

As a result of these properties, apparently ideal for statistical investigation, this m sequence has already been utilized in problems of communications and control technology without special precautionary measures as though these sequences could be used as a substitute for genuine random sequences up to a period length $2^n - 1$. Even expert literature leads to this impression. The present investigation is designed to delineate above all the very narrow limits within which m sequences can substitute for genuine random signals without major concern. This is tied to the warning that beyond these limits, the reliability of the use of pseudorandom signals must be carefully examined in every case.

3. Basic Laws Governing m Sequences

A first indication of the important dependences of the various elements of the m sequence is given in Fig. 2. If we move the m sequence produced in a shift register of length n into a second shift register of length n + 1, and if we tie together the bits corresponding to the generating principle by way of a mod-2 adder, we obtain at the output of this logic a sequence of all zeros. A genuine random statistically independent, evenly distributed binary sequence, however, would yield an evenly distributed random sequence at the output of this network. From the theory of the m sequences (cycle and add property) it is known that in addition to this one

"decoding network" any number of other similar ties can be found which transform the m sequence into a sequence of all zeros. The special characteristics of the pseudorandom signals derived from m sequences discussed here ultimately are based on these fundamental laws in m sequences. As mentioned earlier, pseudorandom sequences are used frequently in statistical tests. The example shows how the test result can degenerate if the test object has a "memory" for more than n bits like the "decoding network" shown in Fig. 2.

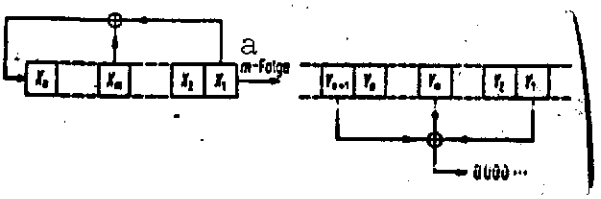

Fig. 2. "Decoding" of an m sequence.

Key: a. m sequence

4. Digital Filtering of m Sequences

An important area of application of m sequences is opened up by linear filtering of m sequences, mainly for the purpose of replacing random signals with a Gaussian distribution [5]. The basic idea is that of summing up a maximum number of uncorrelated, evenly distributed elements of a binary sequence and of approaching a Gaussian process based on the central limit value theory. The summation may also be interpreted as a nonrecursive digital filtering process with low pass filter behavior.

We will now investigate only transverse filters with equal weighting factors (Fig. 3). This involves the direct summation of a finite number of consecutive elements of the m sequence in which the basic laws are particularly stressed.

The pseudorandom signal Z(t) turns out to be a step process whose amplitude can change by no more than one step at consecutive time pulses, depending on whether the total number of logic ones contained in the Y register is increased by one, remains unchanged,

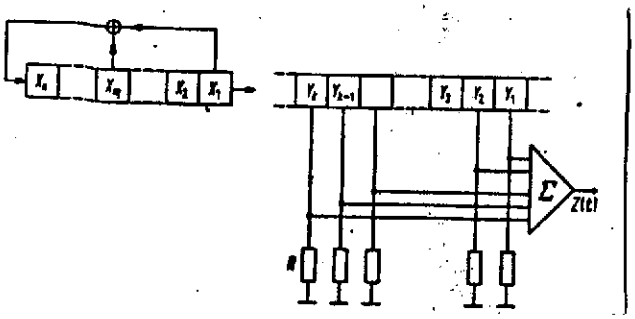

Fig. 3. Summation of a finite number of consecutive bits of the m sequence with a transverse filter.

or is decreased by one. Basic differences in the structure of the output signal Z(t) result as a function of the length of the summation register Y compared with the length of the generating register X. The characteristics of the pseudorandom signal Z(t) are investigated more carefully in terms of amplitude distribution and time-based structure.

4.1. Summation Register Y of Length k ≤ n

The summation register Y corresponds to a window of k storage elements through which the m sequence is shifted. Here, the content of the Y register is identical to bits X_1, X_2, ..., X_k of the generating shift register at an earlier point in time. The total number of the ones which appear under this window in time pulses t_r yields the instantaneous value of the pseudorandom signal $Z(t_r)$. The sequence of these amplitude steps is described by a "pseudorandom variable" $Z = Y_1 + Y_2 + \ldots + Y_k$ which can assume only integral values between zero and the number k of the storage elements taken into account in the summation. The value of the pseudorandom variable Z at clock time t_r is defined by the situation in the summation register Y following r shifts: /32

$$Z(t_r) = Z_r = Y_{1,r} + Y_{2,r} + \cdots + Y_{k,r}, \quad r = 1, 2, \ldots. \tag{1}$$

The initially mentioned property of an m sequence, according to which all possible combinations of zeros and ones with the exception of the zero sequence of length n occur in the generating

X shift register of length n, is frequently significant in the following discussion. Thus, it turns out that for $k \leq n$, within the signal period $2^n - 1$, the amplitude value i occurs exactly $\binom{k}{i}$ times 2^{n-k}, with the exception of the amplitude value zero, which, as a result of the missing zero sequence of length n, has a frequency of $\binom{k}{0} \cdot 2^{n-k} - 1 = 2^{n-k} - 1$.

The relative frequency of the individual amplitude values thus turns out to be

$$H(i) = \frac{\binom{k}{i} 2^{n-k}}{2^n - 1}, \quad H(0) = \frac{2^{n-k} - 1}{2^n - 1} \qquad i = 1, 2, \ldots, k;\ k \leq n. \qquad (2),(3)$$

Variable Z also has an almost exactly binomial distribution, which would also be the case if the binary generating sequence were a genuinely statistically independent sequence in which the zeros and ones occur with an equal degree of probability.

Since the amplitude distribution, as will be shown, gives only insufficient information on whether the pseudorandom signal corresponds to a Gaussian process, we shall consider in addition the time-based signal pattern, particularly the consecutiveness of two amplitude values. In addition, we shall define the relative decreasing frequency $H_A(i)$, the relative increase frequency $H_Z(i)$ and the relative steadiness frequency $H_E(i)$ of an amplitude value i.

The decrease frequency $H_A(i)$ is the relative frequency with which the amplitude value i - 1 follows the amplitude value i. In other words, it can also be said that $H_A(i)$ is the relative frequency with which Z_{r+1} assumes a value of i - 1 provided that Z_r had a value of i:

$$H_A(i) = H(Z_{r+1} < i \mid Z_r = i) = H(Z_{r+1} = i - 1 \mid Z_r = i). \quad (4)$$

The frequency with which an amplitude value i + 1 follows an amplitude value i is the increase frequency $H_Z(i)$:

$$H_Z(i) = H(Z_{r+1} = i + 1 \mid Z_r = i). \quad (5)$$

The steadiness frequency $H_E(i)$, finally, is the frequency with which an amplitude value i follows an amplitude value i:

$$H_E(i) = H(Z_{r+1} = i \mid Z_r = i). \quad (6)$$

Here, the following must apply:

$$H_E(i) + H_A(i) + H_Z(i) = 1. \quad (7)$$

The following discussion is confined at first to the case k < n. The pseudorandom signal assumes the amplitude value i if a total of i ones are located in the summation register Y. A transition to the next smaller amplitude value i - 1 occurs precisely whenever a one is turned over at the output of the summation register and at the same time a zero is taken on at the input.

Provided that a total number of i ones are contained in the Y register, there is a one with the relative frequency i/k at the output of the summation register. Since in the m sequence in n consecutive storage elements, all combinations of zeros and ones occur practically with the same frequency, the taking on of a zero at the input of the summation register is independent of its momentary content if we neglect the missing zero sequence of length n and of relative frequency 1/2, so that for a relative decrease frequency we obtain

$$H_A(i) = \frac{1}{2}\frac{i}{k} \quad (8)$$

The same considerations yield

$$H_Z(i) = \frac{1}{2}\frac{k-i}{k}, \quad H_E(i) = \frac{1}{2}. \qquad (9), (10)$$

The same transition frequencies also result if we use "genuine" statistically independent, evenly distributed binary random sequences.

Entirely different results are obtained if the length of the summation register Y is equal to the length of the generating X register. The bit placed at the input of the Y register is defined clearly in the case of k = n by the content of the Y register. The formation law of the m sequence determines the expression:

$$Y_{n+1} = Y_m \oplus Y_1. \qquad (11)$$

The summation signal always decreases whenever $Y_1 = 1$ and $Y_{n+1} = 0$. Using Eq. (11), these conditions are met exactly whenever $Y_1 = 1$ and $Y_m = 1$.

Under these conditions, of the i ones which lead to the amplitude value i, only i - 2 ones can be randomly distributed across the remaining n - 2 storage elements of the Y registers. Of all the $\binom{n}{i}$ possible combinations in the Y register which yield the amplitude value i, exactly $\binom{n-2}{i-2}$ combinations lead to the decrease of the amplitude value i, provided $Y_1 = 1$ and /33 $Y_m = 1$.

$$H_A(i) = \frac{\binom{n-2}{i-2}}{\binom{n}{i}} = \frac{i(i-1)}{n(n-1)}. \qquad (12)$$

The amplitude value i becomes the amplitude value i + 1 whenever $Y_1 = 0$ and $Y_{n+1} = 1$, or, using Eq. (11), $Y_1 = 0$ and $Y_m = 1$. Under these conditions, the remaining i - 1 ones may be distributed over the remaining n - 2 storage elements in $\binom{n-2}{i-1}$ different ways. We thus obtain for the relative increase frequency

$$H_Z(i) = \frac{\binom{n-2}{i-1}}{\binom{n}{i}} = \frac{i(n-i)}{n(n-1)}. \tag{13}$$

The amplitude value i remains intact provided either $Y_1 = 0$ and $Y_{n+1} = 0$ or $Y_1 = 1$ and $Y_{n+1} = 1$; or, using Eq. (11), $Y_1 = 0$ and $Y_m = 0$ or $Y_1 = 1$ and $Y_m = 0$. This yields a relative steadiness frequency $H_E(i)$:

$$H_E(i) = \frac{\binom{n-2}{i-1}}{\binom{n}{i}} + \frac{\binom{n-2}{i}}{\binom{n}{i}} = \frac{n-i}{n}. \tag{14}$$

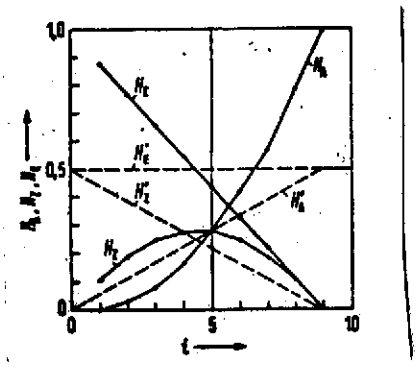

Fig. 4. Comparison of the relative transition frequencies for n = 9, k = 9.

Figure 4 shows $H_A(i)$, $H_Z(i)$ and $H_E(i)$ for n = 9 compared to the corresponding "ideal" frequencies of Eqs. (8) through (10), as would result in a genuine statistically independent binary evenly distributed random sequence as a function of the amplitude value i. The solid curves refer to a pseudorandom signal resulting from filtering an m sequence, generated with a

dual feedback shift register of length n = 9. Here the summation register is also 9 bits long. The dotted curves result from the same filtering of a "genuine" random sequence in accordance with Eqs. (8) through (10). Note the striking differences in the curve patterns of all the transition frequencies as compared to a genuine random process, although in the amplitude distribution and in the power spectrum there is practically no difference as yet. The difference lies uniquely in the time structure of the signals. From the pattern of the steadiness frequency in Fig. 4 we detect, for example, that the pseudorandom process is highly asymmetric with respect to time. If the process occurs at a small amplitude step (i << n), this value is retained for a long period of time, and therefore has a relatively high degree of inertia. If the process occurs at a high amplitude step (i ∿ n), it generally moves on to a lower amplitude step at a rapid pace. Thus, roughly speaking, within high amplitude steps, more frequent variations than within lower amplitude steps occur, an effect which is noted first in the summation of exactly n consecutive bits of the generating m sequence. Figure 5 shows a section from the time sequence $Z_r(t)$ for n = 9 and for a summation length k = 9 in which the asymmetry of the process is clearly visible in the form of pronounced valleys at high amplitude values and wide breakups for small amplitude values.

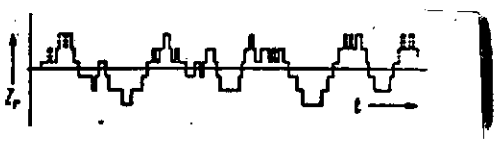

Fig. 5. Section of the time sequence $Z_r(t)$ of the signal for n = 9 with two feedback loops (ℓ = 2) and a summation length k = 9.

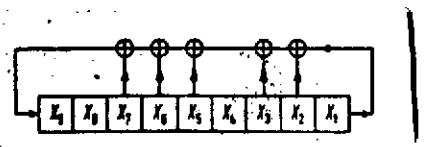

Fig. 6. Example of an m sequence generated by several feedback loops (ℓ = 6).

To obtain an improvement of this asymmetric behavior, m sequences were investigated whose generating principle was

defined by several feedback loops (Fig. 6). These m sequences, for a suitable selection of l feedback loops have the same generally known properties with respect to the period, the autocorrelation function, etc., as the m sequences previously investigated [2].

To calculate $H_A(i)$ we must again make $Y_1 = 1$ and $Y_{n-1} = 0$. This condition can now be met not only by one single given combination of zeros and ones along the l feedback terms, but this condition is always met if an even number of 2j ones is contained in the l feedback terms. Of these possibilities, only those may be considered for the decrease of the amplitude value in which a one is placed on element Y_1. For each j, this yields a total of $\frac{2j}{l}\binom{l}{2j}$ possibilities.

The remaining i - 2j ones may be arranged arbitrarily on the remaining n - l elements, i.e. we obtain for the decrease in amplitude value i for each j a total of $\frac{2j}{l}\binom{l}{2j}\binom{n-l}{i-2j}$ possibilities. /34

Here, for the number of those 2j ones which are placed on the l feedback terms we obtain the trivial limitation of $2j \leq l$ and $2j \leq i$. At the same time, the number of the remaining i - 2j ones may not exceed the number of n - l storage elements of the shift register which do not participate in the feedback process, i.e.

$$i - 2j \leq n - l.$$

To calculate the decrease frequency, we must make a summation over all possible j, while observing these limitations:

$$H_A(i) = \frac{\sum_j \frac{2j}{l}\binom{l}{2j}\binom{n-l}{i-2j}}{\binom{n}{i}} \qquad (15)$$

For the increase frequency we then again obtain conditions $Y_1 = 0$ and $Y_{n+1} = 1$. This is met if the number of ones along the l feedback loops is odd, $2j - 1$, and at the same time, a zero is placed on element Y_1. The remaining $i - (2j - 1)$ ones may again be distributed arbitrarily over the $n - l$ remaining storage elements:

$$H_{\Sigma}(i) = \frac{\sum_{j} \binom{l}{2j-1} \binom{n-l}{i-(2j-1)}}{\binom{n}{i}} \qquad (16)$$

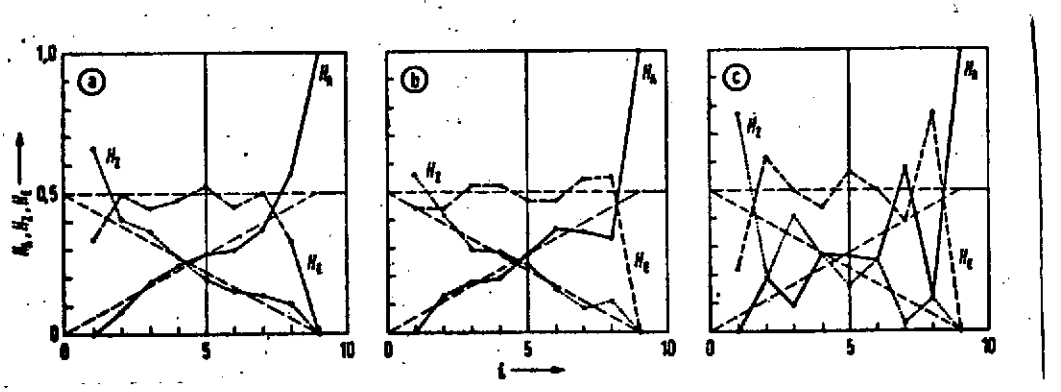

Fig. 7. Relative transition frequencies for (a) n = 9, k = 9 and l = 4. (b) n = 9, k = 9 and l = 6. (c) n = = 9, k = 9 and l = 8.

For the summation index, the same limitations as above result:

$$\left.\begin{aligned} 2j-1 &\leqq i \\ 2j-1 &\leqq l \\ i-(2j-1) &\leqq n-l \end{aligned}\right| \qquad (17)$$

$H_E(i)$ is most easily calculated from the expression

$$H_A(i) + H_T(i) + H_E(i) = 1 \quad (18)$$

The resulting values are shown for n = 9 and τ = 4, 6, 8 in Figs. 7a-c. Compared to Fig. 1, a clear-cut improvement in the transition frequencies of these signals as referred to comparable "genuine" random signals is noted.

In the time periods of the corresponding pseudorandom signals, no such evident asymmetries as those shown in Fig. 5 can be detected. A comparison of Fig. 7a-c shows that an approximation to the ideal conditions, given in Eqs. (8) through (10), is obtained most easily if such m sequences are assumed in which about half the register elements are used for feedback.

4.2. Summation Register Y of Length k > n

For a summation over more than n consecutive elements of the m sequence the amplitude distribtion of the summation signal deviates clearly from a binomial distribution. The following discussion is limited at first again to m sequences which are generated with dual feedback shift registers (Fig. 1). It is shown that the amplitude distribution with increasing summation length becomes highly asymmetric. A quantitative explanation of this phenomenon, using Fig. 8, is at first given for the simplest case of a summation over n + 1 bits. The summation can be carried out in two steps. Initially, the contents of Y_1, Y_m and Y_{n+1} are summed up in the summation register Σ_1, while the other n - 2 register contents are summed up in summation register Σ_2. Since within a time period in the register elements Y_1 to Y_n all combinations of zeros and ones (with the exception of the zero sequence of length n) occur exactly once, /35 an approximately binomially distributed signal occurs at the output of Σ_2. Elements Y_1, Y_m and Y_{n+1} are again mutually

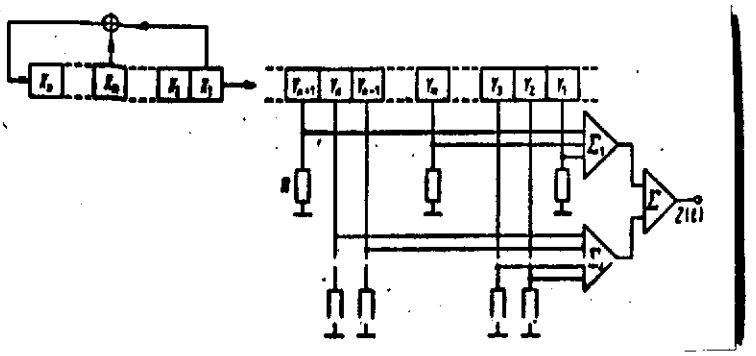

Fig. 8. Summation of n + 1 bits of the m sequence.

tied together as a result of the generating principle of the m sequence, as shown by equation

$$Y_{m+1} = Y_1 \oplus Y_m \qquad (19)$$

Within one time period, all four possible combinations of Y_1 and Y_m occur almost with equal frequency. For each combination, Y_{n+1} can be calculated in accordance with

y_1	y_m	y_{n+1}	Σ_1
0	0	0	0
0	1	1	2
1	0	1	2
1	1	0	2

This also shows that at the output of the summation register Σ_1, only two amplitude values can occur, where amplitude value 2 occurs three times as frequently as amplitude value 0. This asymmetric distribution of the summation signal Σ_1 follows directly from the antivalence Eq. (19) and will therefore be designated the antivalence distribution (Fig. 9). Since all combinations of n consecutive bits (with the exception of the zero sequence) occur exactly once in the m sequence within one time period, there results the amplitude distribution of the output signal Z(t) from the folding of the binomial distribution of Σ_2 with the antivalence distribution of Σ_1, which, according to [9] for example, may occur very simply with the aid of a Z transformation.

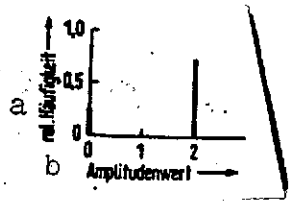

Fig. 9. Amplitude distribution of the summation signal Σ_1 (antivalence distribution).

Key: a. Relative frequency
b. Amplitude value

For a summation over n + 2 bits, the summation is divided into three terms in accordance with the expressions

$$\left.\begin{aligned} Y_{n+1} &= Y_1 \oplus Y_m, \\ Y_{n+2} &= Y_2 \oplus Y_{m+1} \end{aligned}\right| \qquad (20)$$

For the partial summations

and

$$\left.\begin{aligned} \Sigma_1 &= Y_1 + Y_m + Y_{n+1} \\ \Sigma_2 &= Y_2 + Y_{m+1} + Y_{n+2} \end{aligned}\right| \qquad (21)$$

in general we obtain the antivalence distribution of Fig. 9 and for the summation signal across the remaining elements of the summation register we obtain a binomial distribution, but with fewer elements. The amplitude distribution of the entire summation signal is produced again by the folding of the various distributions. Since in this case the antivalence distribution occurs twice, the asymmetry in the summation distribution becomes even more pronounced. Unfortunately this simple calculation method can not be expanded directly to summations of any length, since if Eq. (21) is continued, the same register element will occur in several equations. In this case, a simple antivalence distribution of Fig. 9 must be replaced by a correspondingly complex distribution, which however is generally equally asymmetric. Figure 10 shows the amplitude distribution normalized, as an example, for the standard deviation, for a pseudo-random signal generated by a nine-step shift register and in which the summation stretches over 25 bits.

The time structure of such a summation process is characterized again as in Paragraph 4.1 by the asymmetric transition frequencies. This leads to the fact that the time pattern of the

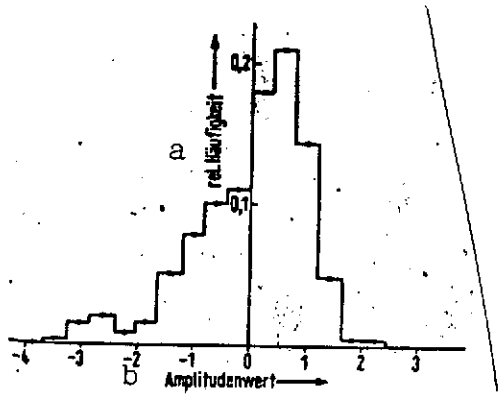

Fig. 10. Schematic of an amplitude distribution for n = 9, k = 25. The m sequence is generated with a dual feedback shift register.

Key: a. Relative frequency
b. Amplitude value

pseudorandom process with increasing summation length is divided into two parts of unequal structure. Most of the time, the process occurs at high amplitudes, and frequently it jumps between two adjacent amplitudes back and forth. In each period, however, the process moves into the domain of a lower amplitude value only once for an extended period of time, exhibiting a completely different time pattern, characterized by the fact that it remains on the same amplitude step for some time and only rarely assumes the adjacent amplitude value.

Tests in which the summation of many consecutive bits of the m sequence was accomplished not with the aid of a transverse filter as shown in Fig. 3, but with a recursive digital fourth-order filter and with a Butterworth characteristic essentially led to the same results. It was shown particularly, in accordance with [7], that an approximately symmetrical amplitude distribution is achieved only if the time window of the low pass filter contains approximately as many bits as correspond to the length of the generating shift register.

If a multiple feedback shift register is used in the generation of the m sequence, complex relationships replace Eq. /36
20. In principle, these expressions also lead to highly asymmetrical amplitude distributions of the summation signal, if the summation occurs over considerably more bits of the m

sequence than the generating shift register length would justify. Figure 11 shows, for example, the amplitude distribution of a pseudorandom signal which was generated by a nine-step shift register with four feedback loops, and in which the summation stretched over 25 bits.

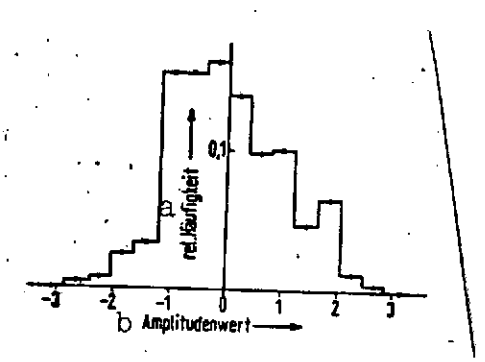

Fig. 11. Schematic of the amplitude distribution for n = 9, k = 25, where the m sequence is generated with a shift register with four feedback loops.

Key: a. Relative frequency
b. Amplitude value

5. Summary

It was shown how the dependences within the m sequence for the summation of consecutive bits of an m sequence become conspicuous in spite of ideal correlation properties. If the summation occurs over fewer bits than the length of the generating shift register, the amplitude distribution and transition frequencies toward the immediately consecutive event coincide practically with an equivalent genuine random signal. If the summation occurs over exactly as many bits as the length of the shift register, the amplitude distribution indeed coincides with that of a correspondingly genuine random signal, but the transition frequencies and the time patterns already show typical deviations. If the summation occurs across at least one bit more than the generating shift register length, the amplitude distributions exhibit typical asymmetries, or in other words: the individual bits of an m sequence, in spite of ideal correlation properties, may be considered at best within the limits of the length of the generating shift register to be statistically independent.

Therefore, tests with m sequences can only be made without concern if no more than n consecutive bits are tied together in the test object or in the evaluation.

(Received June 7, 1972.)

REFERENCES

1. Golomb, S.W., Shift-register sequences, Holden Day, San Francisco, 1967.

2. Peterson, W.W., Prüfbare und korrigierbare Codes [Provable and corrigible codes], R. Oldenbourg Verlag, Munich, 1967.

3. Schweizer, L., "Properties and applications of binary quasi-random sequences," Frequenz 24, 230-234 (1970).

4. Roberts, P.D. and R.H. Davis, "Statistical properties of smoothed maximal-length linear binary sequences," Proc. Inst. Elect. Engrs. 113, 190-196 (1966).

5. Anderson, G.C., B.W. Finnie, and G.T. Roberts,"Pseudorandom and random test signals," Hewlett-Packard J., 2-17, Sept. 1967.

6. Godfrey, K.R., "Theory and application of pseudorandom sequences," Control 10, 305-308 (1966).

7. Gilson, R.P., "Some results of amplitude distribution experiments on shift-register generated pseudorandom noise," Transact. Inst. Elect. Electron. Engrs. EC-15, 926-927 (1966).

8. Matthews, S.B., "Generation of pseudorandom noise having a Gaussian spectral density," Transact. Inst. Elect. Electron. Engrs. EC-17, 382-385 (1968).

9. Davies, A.C., "Probability distributions of noiselike waveforms generated by a digital technique," Electron. Letters 4, 421-423 (1968).

10. Davies, A.C., "Probability-density functions of digitally filtered m sequences," Electron. Letters 5, 222-224 (1969).